

Rotating Strings in $\text{AdS}_4 \times \text{CP}^3$ with B_{NS} holonomy

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ABSTRACT: We study solutions for rigidly rotating strings on $\text{AdS}_4 \times \text{CP}^3$ in the presence of B_{NS} holonomy turned on over $CP^1 \subset CP^3$. We construct general solutions for rotating strings with two and three angular momenta in CP^3 and discuss various limits corresponding to giant magnon and spike like solutions.

KEYWORDS: AdS-CFT correspondence, Bosonic Strings.

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1. Introduction

The proposed ABJM theory [1] has been conjectured to be dual to M-theory on $AdS_4 \times S^7/Z_k$ with N units of four-form flux, which for $k \ll N \ll k^5$ can be compactified to type IIA theory on $AdS_4 \times CP^3$, where k is the level of Chern-Simon (CS) theory with gauge group $U(N) \times U(N)$. In continuation with this proposal, Aharony, Bergman, and Jafferis (ABJ) [2] identified a further class of gauge-gravity duality with extended supersymmetry namely, a three dimensional $N = 6$ superconformal CS theory with a gauge group $U(M)_k \times \overline{U(N)}_{-k}$, with k being the level of the CS theory, is dual to type IIA string theory on $AdS_4 \times CP^3$ with a two form B_{NS} holonomy turned on over $CP^1 \subset CP^3$. In the understanding of the AdS_4/CFT_3 duality better [3, 4, 5, 6, 7, 8, 9, 10], the semiclassical string states in the gravity side have been used to look for suitable gauge theory operators on the boundary. In this connection, the rigidly rotating strings with large angular momentum and energy have been considered and in special limits they correspond to the so called giant magnon [11] and single spike [12] solutions. The corresponding dual field theory with long trace operators with large angular momentum and energy have also been studied in great detail. The generalization of the so called magnon dispersion relation in the presence of two and three angular momenta has also been considered in both asymptotically AdS and non-AdS backgrounds ¹. More recently a class of three spin spiky strings in ABJM model has been studied and various interesting solutions has been obtained in [28].

In this paper, we study the semi-classical string solutions in ABJ model. In [29], it has been shown that ABJ theory also has an integrable structure in planar limit. The rigidly rotating strings corresponding to giant magnon and single spike solution along with the finite size correction has been studied earlier. However the subspace that was considered in [30] was simply $R_t \times S^2$ and hence the solutions correspond to one angular momentum.

¹see for example [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27].

We wish to study the two spin and three spin solutions in the background of $AdS_4 \times CP^3$ in the presence of NS-NS B-field. We have shown that the corresponding solutions obtained in [24, 28] will get contributions due to the presence of the B-field in a very natural way. Further we construct a class of folded string solution in ABJ model and obtain a relation among various conserved charges. The rest of this paper is organized as follows. In section-2, we construct rigidly rotating strings in the background of $AdS_4 \times CP^3$ with NS-NS B-field with two angular momenta on CP^3 . We found two classes of solutions corresponding to giant magnon and spiky strings. We compute all the conserved charges and found out the dispersion relation among them. In section-3, we generalize this solution to three angular momenta on CP^3 . Section-4 is devoted to the study of a class of folded string solutions in ABJ model. Finally in section-5 we conclude with some remarks.

2. Spiky strings with two angular momenta

In this section we study two spin rigidly rotating strings in ABJ model. The metric and NS-NS fields ($B_{\mu\nu}$) for the ABJ model, that is the background of $AdS_4 \times CP^3$ with B_{NS} holonomy is given by

$$\begin{aligned}
ds^2 &= \frac{R^2}{4} \left(-\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\Omega_2^2 \right) + R^2 \left[d\xi^2 + \cos^2 \xi \sin^2 \xi \left(d\psi + \frac{1}{2} \cos \theta_1 d\phi_1 \right. \right. \\
&\quad \left. \left. - \frac{1}{2} \cos \theta_2 d\phi_2 \right)^2 + \frac{1}{4} \cos^2 \xi (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) + \frac{1}{4} \sin^2 \xi (d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2) \right], \\
B_{NS} &= -B \sin 2\xi \, d\xi \wedge d\psi - \frac{B}{2} \sin 2\xi \cos \theta_1 \, d\xi \wedge d\phi_1 + \frac{B}{2} \sin 2\xi \cos \theta_2 d\xi \wedge d\phi_2 \\
&\quad - \frac{B}{2} \cos^2 \xi \sin \theta_1 d\theta_1 \wedge d\phi_1 - \frac{B}{2} \sin^2 \xi \sin \theta_2 d\theta_2 \wedge d\phi_2,
\end{aligned} \tag{2.1}$$

where $0 \leq \xi \leq \frac{\pi}{2}$, $-2\pi \leq \psi \leq 2\pi$ and (θ_i, ϕ_i) are coordinates on two sphere. The radius R is related to 't Hooft coupling constant λ as $R^2 = 2^{\frac{5}{2}} \pi \sqrt{\lambda}$. We are interested in studying the string solutions in a complimentary subspace of $AdS_4 \times CP^3$ for which we take the choice of coordinate in (2.1) as $\rho = 0$ and $\theta_{1,2} = \frac{\pi}{2}$ and $\phi_1 = \phi_2 = \phi$. With the above choice the metric and B field reduces to

$$\begin{aligned}
ds^2 &= -\frac{1}{4} R^2 dt^2 + R^2 [d\xi^2 + \cos^2 \xi \sin^2 \xi d\psi^2 + \frac{1}{4} d\phi^2], \\
B_{NS} &= -B \sin 2\xi \, d\xi \wedge d\psi.
\end{aligned} \tag{2.2}$$

We are interested in studying the solutions of rigidly rotating strings in the above background. We start by writing down the Polyakov action of the fundamental string in the background of (2.2), i.e.

$$S = T \int d\sigma d\tau \left[-\frac{1}{4} (\dot{t}^2 - \dot{t}'^2) + \dot{\xi}^2 - \dot{\xi}'^2 + \cos^2 \xi \sin^2 \xi (\dot{\psi}^2 - \dot{\psi}'^2) + \frac{1}{4} (\dot{\phi}^2 - \dot{\phi}'^2) \right]$$

$$+\frac{B}{4\pi} \int d\sigma d\tau \left[2 \sin 2\xi (\dot{\psi} \xi' - \dot{\xi} \psi') \right], \quad (2.3)$$

where $T = \sqrt{2\lambda}$. The equations of motion derived from the above action (2.3) are

$$\begin{aligned} \sin^2 2\xi (\psi'' - \ddot{\psi}) + \partial_\xi (\sin^2 2\xi) (\xi' \psi' - \dot{\xi} \dot{\psi}) &= 0, \\ \xi'' - \ddot{\xi} - \frac{1}{8} \partial_\xi (\sin^2 2\xi) (\psi'^2 - \dot{\psi}^2) &= 0, \\ \phi'' - \ddot{\phi} &= 0, \\ t'' - \ddot{t} &= 0. \end{aligned} \quad (2.4)$$

To study the rotating string with two angular momenta we take the following ansatz:

$$t = \kappa\tau, \quad \psi = \omega\tau + f(y), \quad \xi = \xi(y), \quad \phi = \nu\tau, \quad (2.5)$$

where we have defined $y = a\sigma + b\tau$. On the other hand, the two Virasoro constraints derived from the action (2.3) are given by

$$\begin{aligned} T_{\tau\tau} + T_{\sigma\sigma} &= \xi_y^2 - \frac{1}{4} \frac{\kappa^2 - \nu^2}{a^2 + b^2} + \frac{1}{4} \sin^2 2\xi (f_y^2 + \frac{\omega^2 + 2b\omega f_y}{a^2 + b^2}) = 0, \\ T_{\tau\sigma} &= \xi_y^2 + \frac{1}{4} \sin^2 2\xi (f_y^2 + \frac{\omega f_y}{b}) = 0. \end{aligned} \quad (2.6)$$

Where ξ_y and f_y denote the partial derivatives of ξ and f with respect to y . The difference of these two Virasoro constraints gives the following relations among various constants,

$$C = -\frac{b}{\omega} (\kappa^2 - \nu^2), \quad (2.7)$$

where C is the integration constant coming from solving the equation of motion for ψ in (2.4) and the addition gives

$$\xi_y = \pm \frac{a\omega}{2(a^2 - b^2) \sin 2\xi} \sqrt{(\cos^2 2\xi_+ - \cos^2 2\xi)(\cos^2 2\xi - \cos^2 2\xi_-)}, \quad (2.8)$$

where $\cos^2 2\xi_\pm =$

$$\frac{2a^2\omega^2 - (\kappa^2 - \nu^2)(a^2 + b^2)}{2a^2\omega^2} \left[1 \pm \sqrt{1 - \frac{(\kappa^2 - \nu^2 - \omega^2)((\kappa^2 - \nu^2)a^2b^2 - a^4\omega^2)}{(2a^2\omega^2 - (\kappa^2 - \nu^2)(a^2 + b^2))^2}} \right]. \quad (2.9)$$

The conserved quantities associated with symmetry of the action (2.3) are

$$E = \pm 2T \frac{\kappa(a^2 - b^2)}{a^2\omega} \int_{\xi_+}^{\xi_-} d\xi \frac{\sin 2\xi}{\sqrt{(\cos^2 2\xi_+ - \cos^2 2\xi)(\cos^2 2\xi - \cos^2 2\xi_-)}},$$

$$\begin{aligned}
J_\phi &= \pm 2T \frac{\nu(a^2 - b^2)}{a^2 \omega} \int_{\xi_+}^{\xi_-} d\xi \frac{\sin 2\xi}{\sqrt{(\cos^2 2\xi_+ - \cos^2 2\xi)(\cos^2 2\xi - \cos^2 2\xi_-)}}, \\
J_\psi &= \pm 2T \frac{a^2 - b^2}{a^2 \omega} \int_{\xi_+}^{\xi_-} d\xi \frac{\sin^3 2\xi}{\sqrt{(\cos^2 2\xi_+ - \cos^2 2\xi)(\cos^2 2\xi - \cos^2 2\xi_-)}} \\
&\quad \left[\omega + \frac{b^2}{\omega(a^2 - b^2)} \left(\omega^2 - \frac{\kappa^2 - \nu^2}{\sin^2 2\xi} \right) \right] - \frac{B}{2\pi} \int_{\xi_+}^{\xi_-} d\xi \, 2 \sin 2\xi.
\end{aligned} \tag{2.10}$$

Taking an infinite volume limit as $\xi_- = \frac{\pi}{4}$, then from (2.9), we have two conditions as (i) $\kappa^2 - \nu^2 = \omega^2$ and (ii) $\kappa^2 - \nu^2 = \frac{a^2}{b^2} \omega^2$. Using the condition (i) we get

$$\kappa E - \nu J_\phi - \omega J_\psi = \left(T + \frac{B}{2\pi} \right) \omega \cos 2\xi_+, \tag{2.11}$$

and the dispersion relation as

$$\sqrt{E^2 - J_\phi^2} - J_\psi = \left(T + \frac{B}{2\pi} \right) \cos 2\xi_+, \tag{2.12}$$

which is a giant magnon like dispersion relation. Using the condition (ii), we get a spike like dispersion relation same as [24], which is

$$\sqrt{E^2 - J_\phi^2} - \frac{1}{2} T \delta\psi = T \left(\frac{\pi}{2} - 2\xi_+ \right), \tag{2.13}$$

with $J_\psi = - \left(T + \frac{B}{2\pi} \right) \cos 2\xi_+$.

3. Rotating strings with three angular momenta

In this section, we generalize the above two spin string solution to three spin string solution in another class of subspace of CP^3 . Here we take the choice of coordinates in (2.1) as $\rho = 0$ and $\theta_{1,2} = \frac{\pi}{2}$. Thus the new metric and B-field look like

$$\begin{aligned}
ds^2 &= -\frac{R^2}{4} dt^2 + R^2 \left[d\xi^2 + \cos^2 \xi \sin^2 \xi d\psi^2 + \frac{1}{4} \cos^2 \xi d\phi_1^2 + \frac{1}{4} \sin^2 \xi d\phi_2^2 \right], \\
B_{NS} &= -B \sin 2\xi \, d\xi \wedge d\psi.
\end{aligned} \tag{3.1}$$

We start by writing down the Polyakov action of the fundamental string in the background (3.1) as

$$\begin{aligned}
S &= -T \int d\sigma d\tau \left[-\frac{1}{4} (t'^2 - \dot{t}^2) + (\xi'^2 - \dot{\xi}^2) + \cos^2 \xi \sin^2 \xi (\psi'^2 - \dot{\psi}^2) + \right. \\
&\quad \left. + \frac{1}{4} \cos^2 \xi (\phi_1'^2 - \dot{\phi}_1^2) + \frac{1}{4} \sin^2 \xi (\phi_2'^2 - \dot{\phi}_2^2) \right]
\end{aligned}$$

$$-\frac{1}{2\pi} \int d\sigma d\tau \ B \sin 2\xi \ (\dot{\xi}\psi' - \xi'\dot{\psi}). \quad (3.2)$$

We choose the following ansatz

$$t = \kappa\tau, \quad \xi = \xi(y), \quad \psi = \omega\tau + f(y), \quad \phi_{i=1,2} = \nu_i\tau + g_i(y), \quad (3.3)$$

where $y = a\sigma + b\tau$ and a, b, κ, ω and ν_i are constants. Then the equations of motion derived from the above action (3.2) is same as [28] as the B-fields are appearing in such a way that their contributions to equations of motion get canceled. The first Virasoro constraint is

$$\begin{aligned} T_{\tau\sigma} &= ab\xi_y^2 + \cos^2 \xi \sin^2 \xi (\omega a f_y + ab f_y^2) + \frac{1}{4} \cos^2 \xi (\nu_1 a g_{1y} + ab g_{1y}^2) \\ &+ \frac{1}{4} \sin^2 \xi (\nu_2 a g_{2y} + ab g_{2y}^2) = 0, \end{aligned} \quad (3.4)$$

and the second virasoro constraint is

$$\begin{aligned} T_{\tau\tau} + T_{\sigma\sigma} &= -\frac{1}{4}\kappa^2 + (a^2 + b^2)\xi_y^2 + \cos^2 \xi \sin^2 \xi [\omega^2 + (a^2 + b^2)f_y^2 + 2\omega b f_y] \\ &+ \frac{1}{4} \cos^2 \xi [\nu_1^2 + (a^2 + b^2)g_{1y}^2 + 2\nu_1 b g_{1y}] \\ &+ \frac{1}{4} \sin^2 \xi [\nu_2^2 + (a^2 + b^2)g_{2y}^2 + 2\nu_2 b g_{2y}] = 0. \end{aligned} \quad (3.5)$$

These Virasoro constraints are same as that given in [28]. Eliminating ξ_y^2 from (3.4) and (3.5) and using results of equation of motion we get the following relation on virasoro constraints

$$\frac{\kappa^2 b}{a^2 - b^2} + 4\omega A_\psi + \nu_1 A_1 + \nu_2 A_2 = 0, \quad (3.6)$$

where A_ψ , A_1 and A_2 are integration constants coming from solutions of equation of motions of ψ , ϕ_1 and ϕ_2 respectively. The conserved conjugate momenta that are associated with the coordinates t, ψ, ϕ_1 and ϕ_2 remains same as in case of ABJM model given in [28] except the conserved quantity that associated with coordinate ψ which gets extra contribution due to the presence of B-field. The conserved momenta are

$$\begin{aligned} E &= 2T \frac{\kappa}{\omega} \frac{a^2 - b^2}{a} \int_{\xi_+}^{\xi_-} d\xi \frac{\sin 2\xi}{\sqrt{(\cos^2 2\xi_+ - \cos^2 2\xi)(\cos^2 2\xi - \cos^2 2\xi_-)}}, \\ J_{\phi_1} &= \left(bA_1 + \frac{1}{2} \frac{\omega_1 a^2}{a^2 - b^2}\right) \frac{E}{\kappa} \mp \frac{T}{2} \frac{\omega_1}{\omega} \int_{\xi_+}^{\xi_-} d\xi \frac{\cos 2\xi \sin 2\xi}{\sqrt{(\cos^2 2\xi_+ - \cos^2 2\xi)(\cos^2 2\xi - \cos^2 2\xi_-)}}, \\ J_{\phi_2} &= \left(bA_2 + \frac{1}{2} \frac{\omega_2 a^2}{a^2 - b^2}\right) \frac{E}{\kappa} \mp \frac{T}{2} \frac{\omega_2}{\omega} \int_{\xi_+}^{\xi_-} d\xi \frac{\cos 2\xi \sin 2\xi}{\sqrt{(\cos^2 2\xi_+ - \cos^2 2\xi)(\cos^2 2\xi - \cos^2 2\xi_-)}}, \end{aligned}$$

$$J_\psi = \left(4bA_\psi + \frac{\omega a^2}{a^2 - b^2}\right) \frac{E}{\kappa} - T \int_{\xi_+}^{\xi_-} d\xi \frac{\cos^2 2\xi \sin 2\xi}{\sqrt{(\cos^2 2\xi_+ - \cos^2 2\xi)(\cos^2 2\xi - \cos^2 2\xi_-)}} - \frac{B}{2\pi} \int_{\xi_+}^{\xi_-} d\xi \sin 2\xi, \quad (3.7)$$

where

$$\cos^2 2\xi_\pm = 1 + \frac{1}{2} \left(\frac{\nu^2 - \kappa^2}{\omega^2} - \frac{b^2 \kappa^2}{a^2 \omega^2} \right) \pm \frac{1}{2\omega^2 \nu a} \left[-64\omega^2 (a^2 - b^2)^2 (\omega^2 + \nu^2) A_\psi^2 - 32\omega^3 b \kappa^2 (a^2 - b^2) A_\psi - 4\omega^2 b^2 \kappa^4 + a^2 \nu^2 \left[\left(1 + \frac{b^2}{a^2}\right) \kappa^2 - \nu^2 \right]^2 \right]^{\frac{1}{2}}, \quad (3.8)$$

and $\nu_1 = \nu_2 = \nu$, $A_2 = A_1 = A$.

The deficit angle is defined as $\delta\phi = \int \frac{\partial\phi}{\partial y} dy$. There is no change in deficit angles as the calculation depends on ξ_y which is derived from virasoro constraints. So they remain unchanged and can be read off from [28].

3.1 Some Cases

Here, we studied some cases where momenta and deficit angles are large or finite. **Case I:** Let us take momenta to be large where as deficit angles are finite. Here we get the relation as

$$E - J_\psi = - \left(T + \frac{B}{2\pi} \right) \sin \delta\psi, \quad (3.9)$$

where $\nu = 0$ and $J_\psi = \frac{\omega E}{\kappa} - \left(T + \frac{B}{2\pi} \right) \cos 2\xi_+$. The dispersion relation is

$$\sqrt{E^2 - J_\phi^2} - J_\psi = - \left(T + \frac{B}{2\pi} \right) \sin \left(\sqrt{1 + \frac{4J^2}{T^2 \pi^2}} \delta\psi \right), \quad (3.10)$$

where $J_\phi = J_{\phi_1} + J_{\phi_2}$, $J = J_{\phi_1} - J_{\phi_2}$, $\sin 2\xi_+ = \frac{b}{a} \sqrt{1 + \frac{\nu^2}{\omega^2}}$, $\frac{\nu}{\omega} = \frac{2J}{T\pi}$.

Case II:

Here we take momenta associated with ϕ_i coordinates as large and finite value for momentum with ψ . We get the dispersion relation

$$\sqrt{E^2 - J_\phi^2} + T\delta\psi = \frac{\kappa^2 - \nu^2}{8\omega^2 J} \pi T^2 (\delta\phi_1 + \delta\phi_2), \quad (3.11)$$

where

$$\delta\psi = - \frac{E}{T} \frac{1}{\sqrt{1 + \frac{\nu^2 b^2}{a^2 \omega^2}}} + \frac{a^2 \omega}{4\nu b^2} (\delta\phi_1 + \delta\phi_2), \quad (3.12)$$

$$J_\psi = - \left(T + \frac{B}{2\pi} \right) \cos 2\xi_+. \quad (3.13)$$

Case III:

Here J_{ϕ_i} 's are finite where as J_ψ is large. The dispersion relation is found to be

$$\frac{E}{\sqrt{1 + \frac{a^2}{b^2} \frac{4J^2}{T^2 \pi^2}}} - J_\psi = \left(T + \frac{B}{2\pi} \right) \cos 2\xi_+, \quad (3.14)$$

where

$$J_\psi = \frac{\omega E}{\kappa} - \left(T + \frac{B}{2\pi} \right) \cos 2\xi_+. \quad (3.15)$$

We can notice that the two spin string solutions in (2.12) and (2.13) with J_ϕ tending to zero give us the one spin giant magnon and a single spike solution. But in case of three spin solutions in (3.10), (3.11) and (3.14), it gives us a new kind of solution.

4. Folded strings

In this section, we wish to study a class of folded string solution in the background of $\text{AdS}_4 \times \text{CP}^3$ in the presence of a B-field. We take the ansatz such that $t, \phi, \psi, \phi_1, \phi_2$ are functions of τ only and $\rho, \theta, \xi, \theta_1, \theta_2$ are periodic functions of σ only. Then the Polyakov action of the string reads as

$$\begin{aligned} S = T \int d\sigma d\tau & \left[\frac{1}{4} \cosh^2 \rho \dot{t}^2 + \frac{1}{4} \sinh^2 \rho (\theta'^2 - \sin^2 \theta \dot{\phi}^2) + \xi'^2 \right. \\ & - \cos^2 \xi \sin^2 \xi (\dot{\psi} + \frac{1}{2} \cos \theta_1 \dot{\phi}_1 - \frac{1}{2} \cos \theta_2 \dot{\phi}_2)^2 + \frac{1}{4} \cos^2 \xi (\theta_1'^2 - \sin^2 \theta_1 \dot{\phi}_1^2) \\ & + \frac{1}{4} \sin^2 \xi (\theta_2'^2 - \sin^2 \theta_2 \dot{\phi}_2^2) \Big] + \frac{B}{4\pi} \int d\sigma d\tau \left[\sin 2\xi \xi' (2\dot{\psi} + \cos \theta_1 \dot{\phi}_1 - \cos \theta_2 \dot{\phi}_2) \right. \\ & \left. + \cos^2 \xi \sin \theta_1 \theta_1' \dot{\phi}_1 + \sin^2 \xi \sin \theta_2 \theta_2' \dot{\phi}_2 \right], \end{aligned} \quad (4.1)$$

The equation of motions for $\rho, \theta, \xi, \theta_1, \theta_2$ are,

$$\begin{aligned} \rho'' &= \sinh \rho \cosh \rho (\dot{t}^2 - \sin^2 \theta \dot{\phi}^2 + \theta'^2), \\ \theta'' &= \sin \theta \cos \theta - 2 \coth \rho \theta' \rho', \\ \xi'' &= -\frac{1}{4} \sin 4\xi (\dot{\psi} + \frac{1}{2} \cos \theta_1 \dot{\phi}_1 - \frac{1}{2} \cos \theta_2 \dot{\phi}_2)^2 + \frac{1}{4} \cos \xi \sin \xi (\sin^2 \theta_1 \dot{\phi}_1^2 - \sin^2 \theta_2 \dot{\phi}_2^2 - \theta_2'^2), \\ \theta_1'' &= 2 \sin^2 \xi \sin \theta_1 (\dot{\psi} + \frac{1}{2} \cos \theta_1 \dot{\phi}_1 - \frac{1}{2} \cos \theta_2 \dot{\phi}_2) \dot{\phi}_1 - \sin \theta_1 \cos \theta_1 \dot{\phi}_1^2 + 2 \tan \xi \xi' \theta_1', \\ \theta_2'' &= -2 \sin^2 \xi \sin \theta_2 (\dot{\psi} + \frac{1}{2} \cos \theta_1 \dot{\phi}_1 - \frac{1}{2} \cos \theta_2 \dot{\phi}_2) \dot{\phi}_2 - \tan^2 \xi \sin \theta_2 \cos \theta_2 \dot{\phi}_2^2 \\ &+ 2 \tan \xi \xi' \theta_2'. \end{aligned} \quad (4.2)$$

Choosing $t = \kappa\tau, \phi = \nu\tau, \psi = \omega\tau, \phi_{i=1,2} = \nu_i\tau$, we get the conserved quantities as

$$E = \frac{T}{2} \pi \kappa \cosh^2 \rho, \quad (4.3)$$

$$J_\phi = \frac{T}{2} \int d\sigma \nu \sinh^2 \rho \sin^2 \theta, \quad (4.4)$$

$$J_\psi = 2T \int d\sigma \cos^2 \xi \sin^2 \xi (\omega + \frac{1}{2} \nu_1 \cos \theta_1 - \frac{1}{2} \nu_2 \cos \theta_2) + \frac{B}{2\pi} \int d\sigma \xi' \sin 2\xi, \quad (4.5)$$

$$J_{\phi_1} = 2T \int d\sigma \left[\frac{1}{4} \nu_1 \cos^2 \xi \sin^2 \theta_1 + \cos^2 \xi \sin^2 \xi \left(\frac{1}{2} \omega \cos \theta_1 + \frac{1}{4} \nu_1 \cos^2 \theta_1 - \frac{1}{4} \nu_2 \cos \theta_1 \cos \theta_2 \right) \right] + \frac{B}{4\pi} \int d\sigma \left[\sin 2\xi \cos \theta_1 \xi' + \cos^2 \xi \sin \theta_1 \theta_1' \right], \quad (4.6)$$

$$J_{\phi_2} = 2T \int d\sigma \left[\cos^2 \xi \sin^2 \xi \left(-\frac{1}{2} \omega \cos \theta_2 - \frac{1}{4} \nu_1 \cos \theta_1 \cos \theta_2 + \frac{1}{4} \nu_2 \cos^2 \theta_2 \right) + \frac{1}{4} \nu_2 \sin^2 \xi \sin^2 \theta_2 \right] + \frac{B}{4\pi} \int d\sigma \left[\sin^2 \xi \sin \theta_2 \theta_2' - \sin 2\xi \cos \theta_2 \xi' \right]. \quad (4.7)$$

The Virasoro constraints remain unchanged as in [31]. For folded string we choose ansatz as $\theta_1 = \theta_2 = 0$. Then the angular momenta changed to

$$\begin{aligned} J_\psi &= 2T \int d\sigma \Theta \cos^2 \xi \sin^2 \xi + \frac{B}{2\pi} \int d\sigma \sin 2\xi \xi', \\ J_{\phi_1} &= T \int d\sigma \Theta \cos^2 \xi \sin^2 \xi + \frac{B}{4\pi} \int d\sigma \sin 2\xi \xi', \\ J_{\phi_2} &= -T \int d\sigma \Theta \cos^2 \xi \sin^2 \xi - \frac{B}{4\pi} \int d\sigma \sin 2\xi \xi', \end{aligned} \quad (4.8)$$

where $\Theta = \omega + \frac{1}{2}(\nu_1 - \nu_2)$ and this gives

$$J_\psi = 2T[K(q) - E(q)] + \frac{B}{2\pi} \sin^2 \xi_0, \quad (4.9)$$

where $K(q)$ and $E(q)$ are complete elliptic integral of the first and second kind with $q = \sin 2\xi_0$. At $\xi_0 = \frac{\pi}{4}$, both E and J 's diverge but gives the dispersion relation

$$E - J_\psi = 2T - \frac{B}{4\pi}. \quad (4.10)$$

5. Conclusions

In this paper we have studied several rigidly rotating string solutions in the background of $\text{AdS}_4 \times \text{CP}^3$ in the presence of B_{NS} holonomy. First we have studied a class of solutions corresponding to the so called giant magnon solution with two spins along a subspace of CP^3 . The corresponding dispersion relation among various charges gets contribution from the B-field. Then we have generalized to three spin solutions in another subspace of CP^3 . We have studied various limiting cases of the solutions and have written down the corresponding dispersion like relations. The finite size corrections to the dispersion relations can be calculated by following [32]. The dual field theory results for the present class of solutions is not completely understood, even though the ABJ theory was shown to be integrable and all string spectrum was unaffected by the presence of discrete B_{NS} holonomy [29]. Further it was shown that the spectrum of all single trace operators is independent of B_{NS} holonomy as well. Hence it appears that the dual operators corresponding to the class of solutions presented here are tractable. The solutions presented here are different from the ones presented earlier is due to the boundary conditions used here. We have used open

string boundary condition which correspond to open spin chain. This fact is essentially responsible for giving a B -dependent term in the dispersion relation among various charges as suggested in [29] from the standpoint of the dual gauge theory. Hence it would really be interesting to investigate the gauge theory dual of the solutions presented here as to see whether the boundary operators gets corrected due to this.

References

- [1] O. Aharony, O. Bergman, D. L. Jafferis, J. Maldacena, “N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals,” *JHEP* **0810**, 091 (2008). [arXiv:0806.1218 [hep-th]].
- [2] O. Aharony, O. Bergman, D. L. Jafferis, “Fractional M2-branes,” *JHEP* **0811**, 043 (2008). [arXiv:0807.4924 [hep-th]].
- [3] J. A. Minahan, K. Zarembo, “The Bethe ansatz for superconformal Chern-Simons,” *JHEP* **0809**, 040 (2008). [arXiv:0806.3951 [hep-th]].
- [4] T. Nishioka, T. Takayanagi, “On Type IIA Penrose Limit and N=6 Chern-Simons Theories,” *JHEP* **0808**, 001 (2008). [arXiv:0806.3391 [hep-th]].
- [5] D. Gaiotto, S. Giombi, X. Yin, “Spin Chains in N=6 Superconformal Chern-Simons-Matter Theory,” *JHEP* **0904**, 066 (2009). [arXiv:0806.4589 [hep-th]].
- [6] G. Grignani, T. Harmark, M. Orselli, “The SU(2) x SU(2) sector in the string dual of N=6 superconformal Chern-Simons theory,” *Nucl. Phys.* **B810**, 115-134 (2009). [arXiv:0806.4959 [hep-th]].
- [7] N. Gromov, P. Vieira, “The all loop AdS4/CFT3 Bethe ansatz,” *JHEP* **0901**, 016 (2009). [arXiv:0807.0777 [hep-th]].
- [8] C. Krishnan, “AdS(4)/CFT(3) at One Loop,” *JHEP* **0809**, 092 (2008). [arXiv:0807.4561 [hep-th]].
- [9] D. Berenstein, D. Trancanelli, “Three-dimensional N=6 SCFT’s and their membrane dynamics,” *Phys. Rev.* **D78**, 106009 (2008). [arXiv:0808.2503 [hep-th]].
- [10] T. McLoughlin, R. Roiban, A. A. Tseytlin, “Quantum spinning strings in AdS(4) x CP**3: Testing the Bethe Ansatz proposal,” *JHEP* **0811**, 069 (2008). [arXiv:0809.4038 [hep-th]].
- [11] D. M. Hofman, J. M. Maldacena, “Giant Magnons,” *J. Phys. A* **A39**, 13095-13118 (2006). [hep-th/0604135].
- [12] M. Kruczenski, “Spiky strings and single trace operators in gauge theories,” *JHEP* **0508**, 014 (2005). [hep-th/0410226].
- [13] N. P. Bobev, H. Dimov, R. C. Rashkov, “Semiclassical strings in Lunin-Maldacena background,” [hep-th/0506063].
- [14] S. Ryang, “Rotating strings with two unequal spins in Lunin-Maldacena background,” *JHEP* **0511**, 006 (2005). [hep-th/0509195].
- [15] C. -S. Chu, G. Georgiou, V. V. Khoze, “Magnons, classical strings and beta-deformations,” *JHEP* **0611**, 093 (2006). [hep-th/0606220].
- [16] M. Kruczenski, J. Russo, A. A. Tseytlin, “Spiky strings and giant magnons on S**5,” *JHEP* **0610**, 002 (2006). [hep-th/0607044].

- [17] J. Kluson, R. R. Nayak, K. L. Panigrahi, “Giant Magnon in NS5-brane Background,” JHEP **0704**, 099 (2007). [hep-th/0703244].
- [18] R. Ishizeki, M. Kruczenski, “Single spike solutions for strings on S^{*2} and S^{*3} ,” Phys. Rev. **D76**, 126006 (2007). [arXiv:0705.2429 [hep-th]].
- [19] N. P. Bobev, R. C. Rashkov, “Spiky strings, giant magnons and beta-deformations,” Phys. Rev. **D76**, 046008 (2007). [arXiv:0706.0442 [hep-th]].
- [20] H. Dimov, R. C. Rashkov, “On the anatomy of multi-spin magnon and single spike string solutions,” Nucl. Phys. **B799**, 255-290 (2008). [arXiv:0709.4231 [hep-th]].
- [21] B. -H. Lee, R. R. Nayak, K. L. Panigrahi, C. Park, “On the giant magnon and spike solutions for strings on $AdS(3) \times S^{*3}$,” JHEP **0806**, 065 (2008). [arXiv:0804.2923 [hep-th]].
- [22] J. R. David, B. Sahoo, “Giant magnons in the D1-D5 system,” JHEP **0807**, 033 (2008). [arXiv:0804.3267 [hep-th]].
- [23] B. -H. Lee, K. L. Panigrahi, C. Park, “Spiky Strings on $AdS(4) \times CP^{*3}$,” JHEP **0811**, 066 (2008). [arXiv:0807.2559 [hep-th]].
- [24] S. Ryang, “Giant Magnon and Spike Solutions with Two Spins in $AdS(4) \times CP^{*3}$,” JHEP **0811**, 084 (2008). [arXiv:0809.5106 [hep-th]].
- [25] M. C. Abbott, I. Aniceto, “Giant Magnons in $AdS(4) \times CP^{*3}$: Embeddings, Charges and a Hamiltonian,” JHEP **0904**, 136 (2009). [arXiv:0811.2423 [hep-th]].
- [26] R. Suzuki, “Giant Magnons on CP^{*3} by Dressing Method,” JHEP **0905**, 079 (2009). [arXiv:0902.3368 [hep-th]].
- [27] M. C. Abbott, I. Aniceto, O. Ohlsson Sax, “Dyonic Giant Magnons in CP^3 : Strings and Curves at Finite J ,” Phys. Rev. **D80**, 026005 (2009). [arXiv:0903.3365 [hep-th]].
- [28] S. Giardino, H. L. Carrion, “Classical strings in $AdS(4) \times CP^{*3}$ with three angular momenta,” [arXiv:1106.5684 [hep-th]].
- [29] D. Bak, D. Gang, S. -J. Rey, “Integrable Spin Chain of Superconformal $U(M) \times$ anti- $U(N)$ Chern-Simons Theory,” JHEP **0810**, 038 (2008). [arXiv:0808.0170 [hep-th]].
- [30] S. Jain, K. L. Panigrahi, “Spiky Strings in $AdS(4) \times CP^{*3}$ with Neveu-Schwarz Flux,” JHEP **0812**, 064 (2008). [arXiv:0810.3516 [hep-th]].
- [31] B. Chen, J. -B. Wu, “Semi-classical strings in $AdS(4) \times CP^{*3}$,” JHEP **0809**, 096 (2008). [arXiv:0807.0802 [hep-th]].
- [32] G. Arutyunov, S. Frolov, M. Zamaklar, “Finite-size Effects from Giant Magnons,” Nucl. Phys. **B778**, 1-35 (2007). [hep-th/0606126].